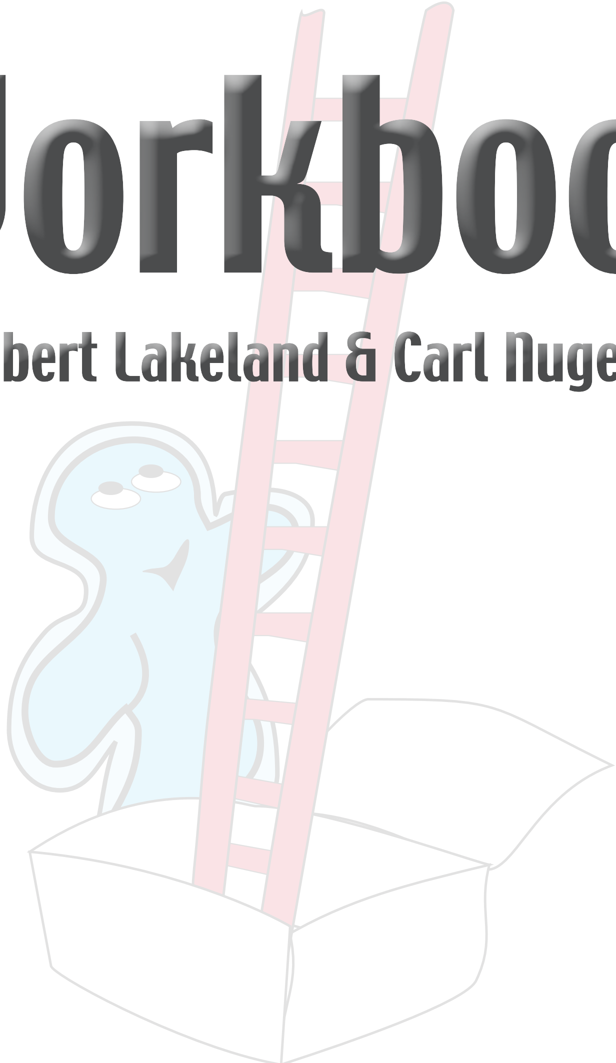
Year 11

Mathematics

EAS

Workbook

Robert Lakeland & Carl Nugent



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Contents

Algebra – Adding and Subtracting terms

Algebra (+, –)

Algebraic terms comprise numbers and variables multiplied together. Sometimes the variables are raised to a power or exponent, e.g. $4x^2$

Power or exponent of x is 2.

$4x^2$

Coefficient is 4

Variable is x

To add or subtract algebraic terms they must have the same variables with the same power or exponent. Such terms are called **like terms**.

e.g. $2x, 5x, {}^{-}8x, 6x, \frac{1}{2}x$ etc. are like terms as the only variable is x and it is raised to the power of 1 (implied).

$4x^2y, 5x^2y, {}^{-}6x^2y, 9x^2y$ etc. are like terms as all have x to the power of 2 and y to the power of 1.

$5a, {}^{-}3a, 2a, 12a$ etc. are like terms

When adding or subtracting algebraic terms add or subtract the **like** terms together.

Extra Notes

Begin by identifying the like terms by drawing a shape around them and then add or subtract the ones with the same shape. Draw the shape around the + or – in front of the term as well.

Algebraic terms without a number in front imply a coefficient of 1, e.g. x implies 1x, ab implies 1ab. Accepted practice is that if the number in front of the variable is a '1 ', we leave it out.

The algebraic term ab is equivalent to the term ba, but it is conventional to write the letters in alphabetical order.

Example

Simplify $4a^2 – 5b + 9 – 7a^2 + 10b – 11$

We begin by identifying the like terms, a^2, b and constant terms.

$$4a^2 \; – 5b \; + 9 \; – 7a^2 \; + 10b \; – 11$$

Combining like terms taking into account the sign in front of each term.

a^2 terms $\quad 4a^2 – 7a^2 = {}^{-}3a^2$

b terms $\quad – 5b + 10b = 5b$

Constant terms $\quad + 9 – 11 = {}^{-}2$

So

$$4a^2 – 5b + 9 – 7a^2 + 10b – 11 = {}^{-}3a^2 + 5b – 2$$

Example

Simplify $3x^2 – 9xy + 6x^2 + 8xy – 5$

We begin by identifying the like terms and their sign.

$$3x^2 \; – 9xy \; + 6x^2 \; + 8xy \; – 5$$

Combining like terms taking into account the sign in front of each term.

x^2 terms $\quad 3x^2 + 6x^2 = 9x^2$

xy terms $\quad – 9xy + 8xy = {}^{-}xy$

Constant terms $\quad – 5 = {}^{-}5$

So

$$3x^2 – 9xy + 6x^2 + 8xy – 5 = 9x^2 – xy – 5$$

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Example

Solve $3(x + 2) = {}^-9$

$$
\begin{aligned}
& & 3(x+2) &= {}^-9 \\
&\text{Expanding} & 3x + 6 &= {}^-9 \\
&\text{Sub. 6 from both sides} & 3x + 6 - 6 &= {}^-9 - 6 \\
&\text{Simplifying} & 3x &= {}^-15 \\
&\text{Dividing by 3} & \frac{3x}{3} &= \frac{{}^-15}{3} \\
&\text{Simplifying} & x &= {}^-5
\end{aligned}
$$

Example

Solve $4x - 7 = 6x + 13$

$$
\begin{aligned}
& & 4x - 7 &= 6x + 13 \\
&\text{Adding 7 to both sides} & 4x - 7 + 7 &= 6x + 13 + 7 \\
&\text{Simplifying} & 4x &= 6x + 20
\end{aligned}
$$

Subtract 6x from both sides so all the variables are on the left side. $4x - 6x = 6x + 20 - 6x$

$$
\begin{aligned}
&\text{Simplifying} & {}^-2x &= 20 \\
&\text{Dividing by } {}^-2 & \frac{{}^-2x}{{}^-2} &= \frac{20}{{}^-2} \\
&\text{Simplifying} & x &= {}^-10
\end{aligned}
$$

Example

Solve $\frac{3x}{5} = \frac{x+2}{3}$

We begin removing the fractions by multiplying each side by 15, (the common multiple of 5 and 3).

$$
\begin{aligned}
& & \frac{3x}{5} &= \frac{x+2}{3} \\
&\text{Multiply both sides by 15} & \frac{3x}{5} \times 15 &= \frac{x+2}{3} \times 15 \\
&\text{Simplifying} & 3x \times 3 &= (x+2) \times 5 \\
& & 9x &= 5x + 10 \\
&\text{Simplifying} & 4x &= 10 \\
&\text{Dividing by 4} & \frac{4x}{4} &= \frac{10}{4} \\
&\text{Simplifying} & x &= 2.5
\end{aligned}
$$

Example

Solve $\frac{3x}{2} - 4 = \frac{1}{2}$

We begin removing the fractions by multiplying every term by 2.

$$\frac{3x}{2} - 4 = \frac{1}{2}$$

Multiplying every term by the common multiple of the denominators (2). $\frac{3x}{2} \times 2 - 4 \times 2 = \frac{1}{2} \times 2$

$$
\begin{aligned}
&\text{Simplifying} & 3x - 8 &= 1 \\
&\text{Add 8 to both sides} & 3x - 8 + 8 &= 1 + 8 \\
&\text{Simplifying} & 3x &= 9 \\
&\text{Dividing by 3} & \frac{3x}{3} &= \frac{9}{3} \\
&\text{Simplifying} & x &= 3
\end{aligned}
$$

Expanding

Expanding

The process of **expanding** is to remove brackets from an algebraic expression.

The two basic types of problems you need to deal with are those where there is a single term outside the brackets

e.g. $3x(x-2)$

and those where there are two bracketed terms

e.g. $(2x+1)(3x-4)$

If there is only one bracketed term and a single term in the front or back of the bracketed term, then the terms inside the brackets are multiplied by the single term outside.

If there are two bracketed terms, the procedure is to multiply each term in one bracket by each term in the other bracket and then group together the terms that are **like**.

The expansion of both types is represented diagrammatically below. The arrows indicate the required multiplications.

$$\begin{aligned} 3x(x-2) &= 3x(x-2) \\ &= 3x \times x - 3x \times 2 \\ &= 3x^2 - 6x \end{aligned}$$

For $(2x+1)(3x-4)$ we multiply the second bracket by each part of the first bracket.

$$\begin{aligned} (2x+1)(3x-4) &= 2x \times (3x-4) + 1 \times (3x-4) \\ &= 2x \times 3x - 2x \times 4 + 1 \times 3x - 1 \times 4 \\ &= 6x^2 - 8x + 3x - 4 \\ &= 6x^2 - 5x - 4 \qquad \text{Simplifying} \end{aligned}$$

We can use the arrows to restate this process as

$$\begin{aligned} (2x+1)(3x-4) &= (2x+1)(3x-4) \\ &= 2x \times 3x - 2x \times 4 + 1 \times 3x - 1 \times 4 \\ &= 6x^2 - 8x + 3x - 4 \\ &= 6x^2 - 5x - 4 \qquad \text{Simplifying} \end{aligned}$$

When we have a squared bracket we restate this and use the process above.

$$\begin{aligned} (3x+2)^2 &= (3x+2)(3x+2) \\ &= 3x \times 3x + 3x \times 2 + 2 \times 3x + 2 \times 2 \\ &= 9x^2 + 6x + 6x + 4 \\ &= 9x^2 + 12x + 4 \end{aligned}$$

Extra Notes

When asked to expand an expression always simplify after the expansion, if possible.

When asked to expand a squared bracket such as $(x-a)^2$ always write in the form $(x-a)(x-a)$ before expanding.

The Perfect Square pattern

Two brackets squared (e.g. $(x+9)^2$) is called a perfect square. When we multiply out a perfect square the result follows a pattern.

$$(x \pm a)^2 = x^2 \pm 2ax + a^2$$

OR $$(cx \pm a)^2 = c^2x^2 \pm 2acx + a^2$$

We can use this pattern to expand perfect squares by inspection by looking for the value of 'a' and substituting it in the pattern.

e.g. $(x-9)^2 = x^2 - 18x + 81$

a = 9 so $a^2 = 81$

Signs are the same

a = 9 so 2a = 18

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192. $(x + 5)(2x - 5)$

193. $(2x + 1)(3x - 2)$

194. $2(x + 5)(x - 2)$

195. $3(2x + 1) + 4(x - 2)$

196. $4(3x - 1) - 2(x - 4)$

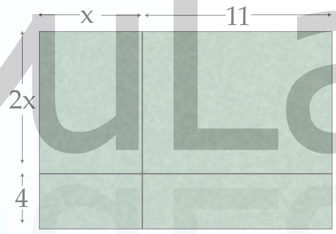
197. $x(3x - 2) + 2x$

198. $^{-}4x(2x - 3) + x^2 - 5$

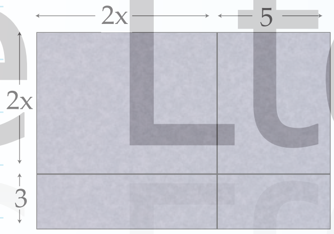
199. $5(2x - 1)(3x + 4)$

Merit – Form an expression and use it to find an expanded expression for the area of each figure.

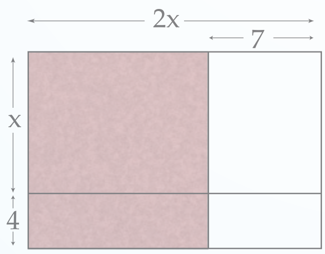
200. Find the shaded area in expanded form.



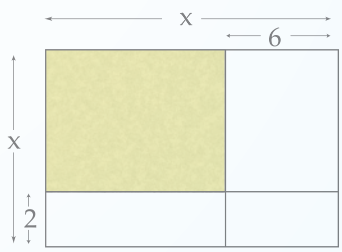
201. Find the shaded area in expanded form.



202. Find the shaded area in expanded form.



203. Find the shaded area in expanded form.



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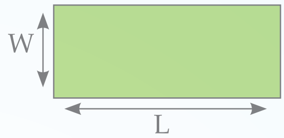
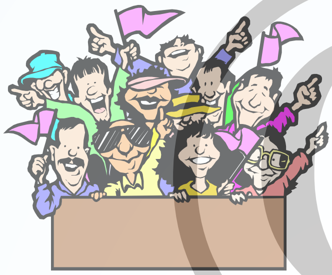
401. For $140 a sports fan can purchase 2 seats in the grandstand and 4 seats on the terraces. For an additional $55 one extra seat in the grandstand and one extra seat on the terraces can be purchased. What is the unit price of seats in the grandstand and on the terraces?

402. The length of a rectangle is 4 times its width. If the rectangle's perimeter is 280 cm. What are the dimensions of the rectangle?





403. A publisher produces two types of Workbooks. If one school buys 6 copies of the Calculus Workbook and 3 copies of the Year 11 Workbook and pays $195, while another school purchases 2 copies of the Calculus Workbook and 4 copies of the Year 11 Workbook and pays $140, what is the unit cost of each Workbook?

404. Tickets to a play cost $20 for adults and $5 for students. 500 tickets were sold for $7000. How many adults and students attended?

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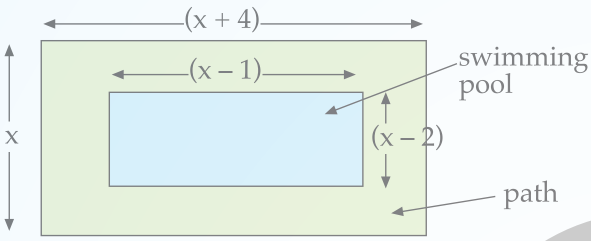
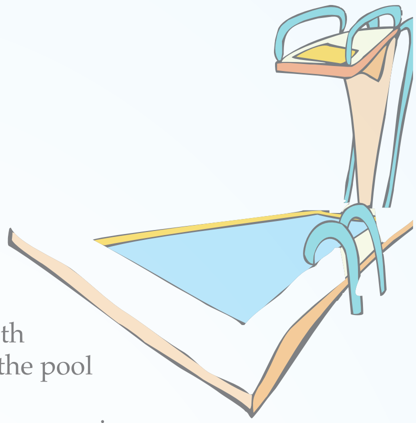
405. A dairy sells $3 and $5 ice-creams. In one day they sell 50 ice-creams earning a total of $180. How many of each type of ice-cream does the dairy sell?

406. A multi-choice test comprises 35 questions. Some of the questions are worth 2 marks and others 3 marks. If the test is out of 90 marks how many of each type of questions are there?

433. A rectangular swimming pool is situated in a backyard and is surrounded by a path (see the diagram below). All measurements are in metres, but NOT to scale.





a) Find an expression for the entire area (path plus swimming pool).
Remember to expand and simplify your answer.

b) Find an expression for the area of the swimming pool only (expand and simplify your answer).

c) It is known that the area of the path surrounding the pool occupies 54 m^2, find an expression for just the area of the path surrounding the swimming pool and hence calculate the dimensions of the pool.
Explain what you are doing.

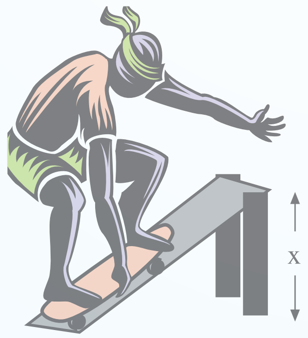
434. Shanti is experimenting with an adjustable ramp as a skateboard jump.
She alters the height to see how this affects the distance she travels horizontally in a jump.

Shanti has modelled the distance of the jump with the formula

Distance = $x(k - x)$

where x is the vertical height at the end of the ramp in centimetres and k is a constant that she needs to work out. Shanti knows that when $x = 10$ the jump distance was 280 cm.

For a competition Shanti wants to jump exactly 240 cm. Form an equation in x only and solve it to find the height she should set the ramp to, to make the required jump as spectacular as possible while still landing 240 cm from the jump.



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Linear Graphs by Plotting Points

Plotting Points

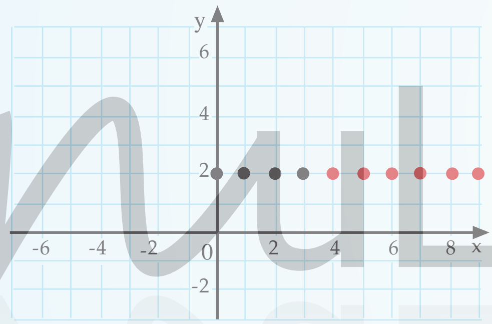
If we have any algebraic relation, we can draw a graph by plotting points and joining them up.

The straight line $y = 2$ always has the y ordinate (the y part of each ordered pair) equal to 2 no matter what the x ordinate is equal to.

Therefore a table of points could be:

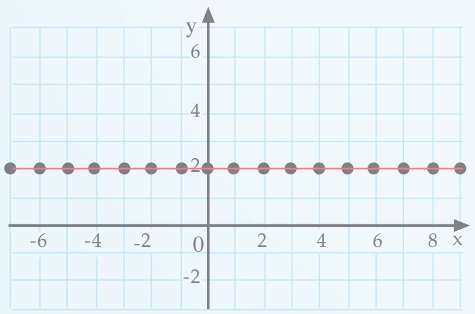
x	y
0	2
1	2
2	2
3	2

These coordinates (in black) are plotted on the graph. If the relationship was defined just for whole numbers {0, 1, 2, 3, ...} then the points would be extended to show this (in red).



Similarly if the relationship was defined for integers they would be individually plotted (not joined up - see below).

If the equation represented real numbers we would join up all the points to show this and extend the line (see below - in red).



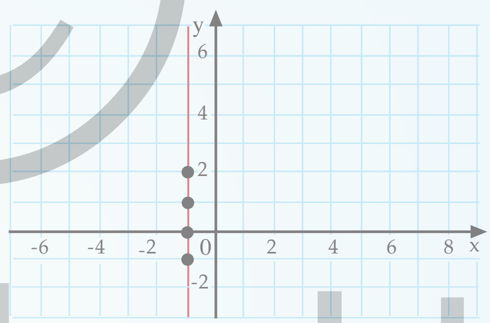
We can generalise from this graph:

Any line in the form $y = k$ is a horizontal line passing through k on the y axis.

The straight line $x = {}^{-}1$ always has the x ordinate (the x part of each ordered pair) equal to $^{-}1$ no matter what the y ordinate is equal to. Therefore a table of points would be:

x	y
$^{-}1$	$^{-}1$
$^{-}1$	0
$^{-}1$	1
$^{-}1$	2

These coordinates are plotted on the graph below and joined to show the relation $x = {}^{-}1$ for real numbers.

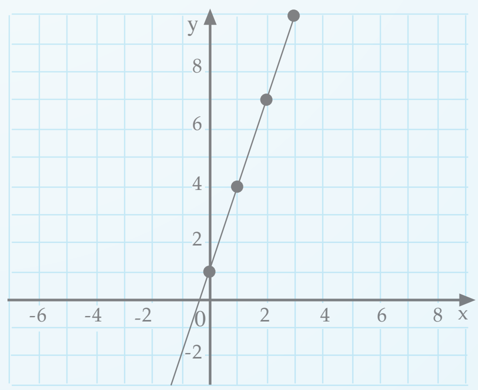


Any line in the form $x = k$ is a vertical line passing through k on the x axis.

For the relation $y = 3x + 1$ we select any four x values and substitute each x value in the equation to find the corresponding y value.

x	3x – 1 = y
0	3 x 0 + 1 = 1
1	3 x 1 + 1 = 4
2	3 x 2 + 1 = 7
3	3 x 3 + 1 = 10

These coordinates are plotted on the graph below and joined to show the relation $y = 3x + 1$ for real numbers.



96. The volume of water in a reservoir is represented by the equation

$V = {}^{-}245t + 18\,200$

where t is the time in hours and V the volume in litres.

a) Find the rate of change of volume in litres per hour.

b) How long will it take until the reservoir is empty?

97. When a metal bar is heated it expands and when cooled contracts. The length of a metal bar, L (cm) when heated to a temperature t (°C) is represented by the equation

$L = 0.0167t + 120$

a) Find the rate of change of length in centimetres per °C.

b) To what temperature would the bar have to be heated to reach a length of 123 cm?

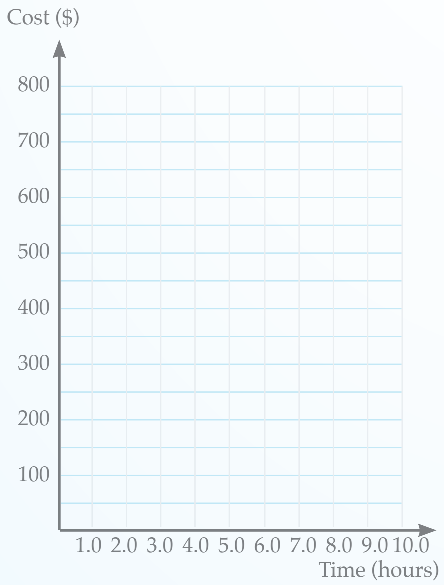
98. The production time and cost of five different products produced by a company are recorded in the table below.

a) Graph the tabulated results on the grid below.

b) Calculate the rate of change of the cost.

c) Give the equation relating the cost (C) and time (T).

Time (hours)	Cost ($)
3.5	330
4.0	370
6.5	570
7.0	610
9.0	770

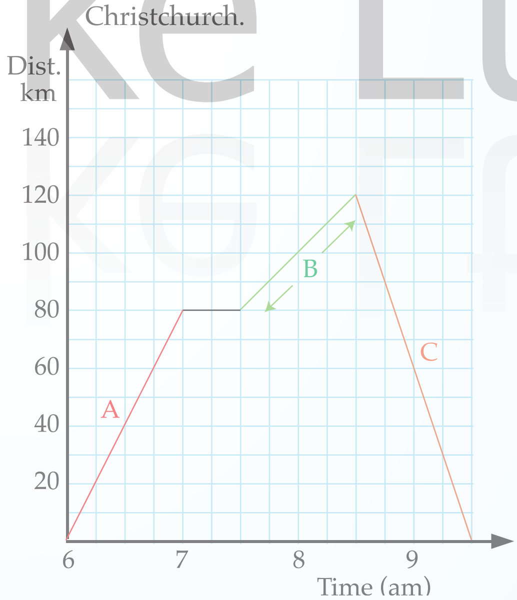


99. The graph below represents the distance a car has travelled after leaving Christchurch.

a) Find the rate of change for each of the three sections A, B, and C.

b) Give the equation that represents section A of the journey.

c) Using your equation from b) find at what time the car is 48 km from Christchurch.



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Linear Patterns

Linear Patterns

Linear patterns are sequences of numbers where the difference between successive terms is always the same.

For example, the sequence of terms

4, 7, 10, 13, 16, 19, 22

has a common difference of 3. It has a linear pattern.

If a sequence has a common difference d, then the rule that generates that sequence is going to start

term = dn ± a constant

We use n rather than x as n represents natural numbers {1, 2, 3, 4, ...}. x is used for real numbers which have no meaning in a sequence.

The sequence 4, 7, 10, 13, 16, 19, 22 is generated by the rule

term = 3n ± a constant

By substituting any term we can find the constant.

The 1st term is 4, i.e. (1, 4) so substituting

4 = 3 x 1 ± a constant

gives the constant as + 1. The rule that generates the sequence is

term = 3n + 1

By substituting in values for n the sequence 4, 7, 10, 13, 16, 19, 22 is generated.

Example

A table at a timber yard relates the length of a fence to the amount of 100 by 50 railing timber required.

Fence length (m)	Timber length (m)
1	7
2	11
3	15
4	19
5	23

a) Find the rule that relates the amount of timber to the length of fence.

b) How much timber is required for a fence that is 21 metres long?

a) The common difference between terms is 4, so the rule starts

Timber = 4 x fence length ± constant

By substituting (1, 7) we find the constant is 3, so

Timber = 4 x fence length + 3

b) By substituting 21 metres into our rule i.e. Timber = 4 x 21 + 3

we get the amount of timber required which is 87 metres.

If you can identify the term 'previous' to the first term then the formula becomes term = dn + previous.

For the sequence 4, 7, 10, 13, ... the term previous to the first term 4, is 1, so the rule is term = 3n + 1.

You can use a graphics calculator to find the rule for a linear pattern.

On the Casio 9750GII from the MENU select STAT then clear any values in List 1 and List 2 by selecting more, delete all and yes.

▷ [F6] DEL-A [F4] Yes [F1]

Place the term numbers in List 1 (i.e. 1, 2, 3, 4 etc.) and the sequence values in List 2 (i.e. 4, 7, 10, 13 etc.).

Select GRPH [F1] and then SET [F6].

Make sure you have specified:

Graph Type: Scatter, XList: List1, YList: List2,

Frequency: 1

Press [EXE]

Now select GPH1 [F1] to draw the line of your pattern.

Press CALC [F1], X [F2] and aX + b [F1] to get the a and b values for the linear pattern which is of the form y = ax + b.

On the TI-84 Plus press [STAT] [1]

and delete any existing columns by moving the cursor to the top of the column and pressing [CLEAR] [ENTER]

Place the term numbers in L1 (i.e 1, 2, 3, 4 etc.) and the sequence values in L2.

Select [STAT] [CALC] [4] [ENTER] to get the a and b values for the linear pattern which is of the form y = ax + b.

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Example

Graph the following parabolas

a) $y = x^2 - 2$ b) $y = (x + 5)^2$ c) $y = (x - 2)^2 + 3$

a) For $y = x^2 - 2$

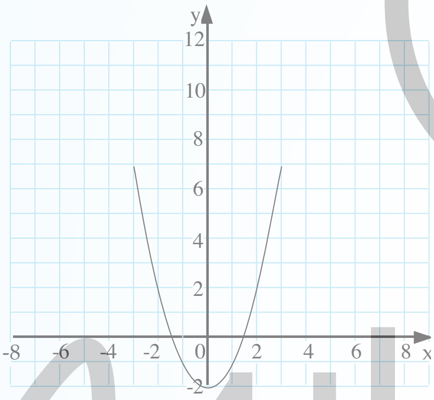
The base of the graph of $y = x^2$ has been shifted down 2. From this point we go across 1 up 1, across 1 up 3, across 1 up 5, across 1 up 7 etc.

b) For $y = (x + 5)^2$

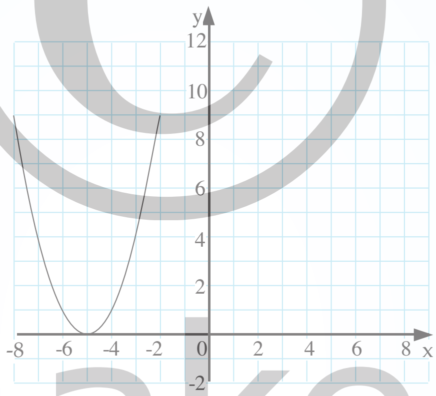
The base of the graph of $y = x^2$ has been shifted sideways to $^-5$. From this point we go across 1 up 1, across 1 up 3, across 1 up 5, across 1 up 7 etc.

c) For $y = (x - 2)^2 + 3$

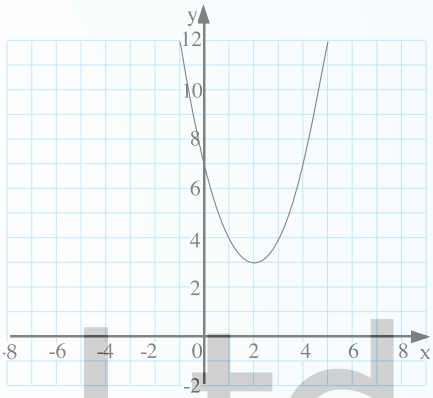
The base of the graph of $y = x^2$ has been shifted sideways 2 and up 3. From this point we go across 1 up 1, across 1 up 3, across 1 up 5 etc.



$y = x^2 - 2$



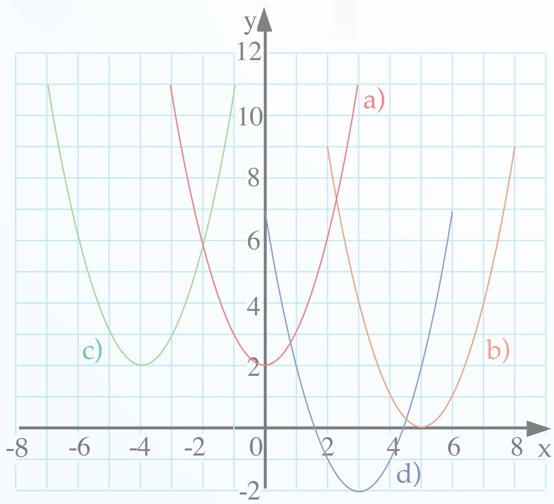
$y = (x + 5)^2$



$y = (x - 2)^2 + 3$

Example

Write the equation of each parabola drawn on the grid below.



a) $y = x^2 + 2$

As the graph $y = x^2$ is moved up 2 units.

b) $y = (x - 5)^2$

As the graph $y = x^2$ is moved right 5 units.

c) $y = (x + 4)^2 + 2$

As the graph $y = x^2$ is moved left 4 units and up 2 units.

d) $y = (x - 3)^2 - 2$

As the graph $y = x^2$ is moved right 3 units and down 2 units.

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b) Complete the table below for the bacterial population over a seven hour period.

Term	Population	1st Diff.	Ratio of 1st Differences
1			
2			
3			
4			
5			
6			
7			

c) Find the rule that relates the bacterial population, P, after h hours.

d) Using your rule from part (c) find the bacterial population after 10 hours.

e) By substituting values for h, in your equation, find out after how many hours the bacterial population will exceed 1 000 000.

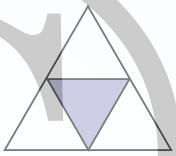
225. A fractal is 'a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole,' a property called self-similarity.

A well known fractal is based on a triangle shape called the Sierpinski triangle.

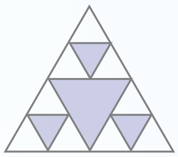
Stage 1 is taking the midpoint of the three sides of an unshaded triangle, connecting them and shading the new triangle formed.

Stage 2 repeats the process for each of the unshaded triangles.

See the diagram below.



Stage 1



Stage 2

a) Draw stage 3 of the Sierpinski triangle fractal below.

Stage 3

b) Complete the table below for the number of unshaded triangles at each stage of the Sierpinski triangle.

Stage	Unshaded triangles	1st Diff.	Ratio of 1st Differences
1			
2			
3			
4			
5			
6			
7			

c) Find the rule that relates the number of unshaded triangles, U to the stage number, s.

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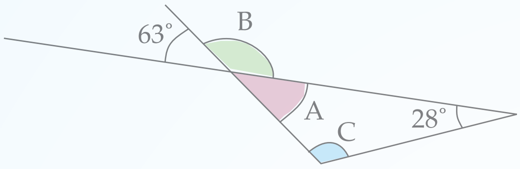
Example

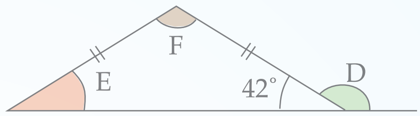
Find, with reasons, each of the unknown angles.



A	= 63°	Vertically opposite angles are equal.
B	= 117°	Adjacent angles on a straight line = 180°.
28 + 63 + C	= 180	Angle sum of a Δ = 180°.
C	= 89°	

Example

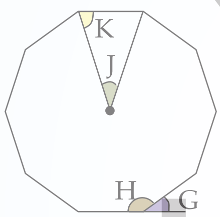
Find, with reasons, each of the unknown angles.



D	= 138°	Adjacent angles on a straight line = 180°.
E	= 42°	Base angles of an isos. triangle are =.
42 + 42 + F	= 180	Angle sum of a triangle is 180°.
F	= 96°	

Example

Find, with reasons, each of the unknown angles in this regular decagon.



G	$= \frac{360}{10}$	Exterior angles of
G	= 36°	a polygon = 360°.
H	= 144°	Adjacent angles on a straight line = 180°.
J	$= \frac{360}{10}$	Angle sum of a point = 360°.
	= 36°	All ten angles are equal.
K	$= \frac{180 - 36}{2}$	Base angles of an isosceles
	= 72°	triangle are equal.

The REASON for an answer is the geometric property and not the arithmetic that led to the answer.

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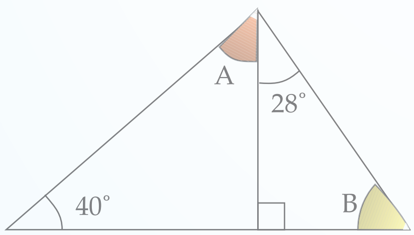
Achievement – Calculate each required angle. An answer with no geometric reason is only achievement.

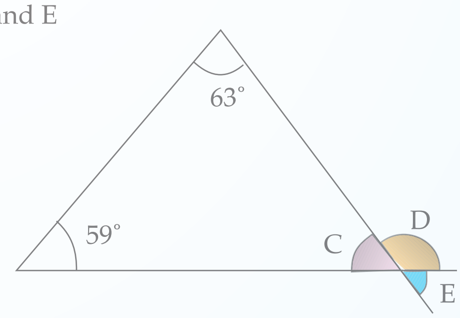
Merit – Find the required angles along with the geometric justification (reason).

1. A and B.



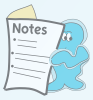
Calculations *Geometric reason*

2. C, D and E



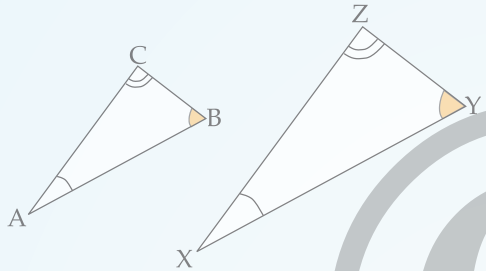
Calculations *Geometric reason*

Similar Triangles



Similar Triangles

When two triangles are equiangular then one triangle is an enlargement of the other.



As one triangle is an enlargement of the other then the ratio of any side on one triangle to its equivalent side on the other triangle is the same.

e.g. $\dfrac{AB}{XY} = \dfrac{BC}{YZ}$

With any pair of similar triangles the equivalent angles of the triangles are equal. Once we have identified a pair of similar triangles we can use the constant ratio between the sides to solve problems.

Procedure. Get the two triangles so that their equivalent corners are in the same relative position. For triangles $\triangle ABC$ we know that the angle equal to A on the second triangle is X, the angle equal to B is Y and the angle equal to C is Z . Therefore the triangle similar to $\triangle ABC$ is $\triangle XYZ$ (The letters describing the second triangle are in the same relative position).

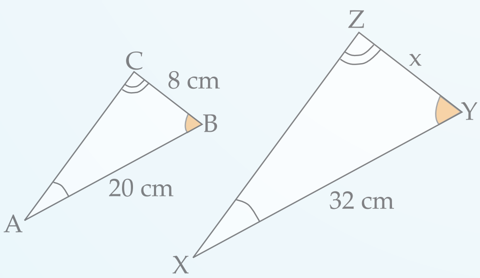
Once we have identified our similar triangles,

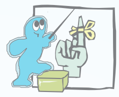
$$\text{similar triangles} = \frac{\triangle XYZ}{\triangle ABC}$$

we can see that Y is above B and Z is above C so YZ is an enlargement of BC.

$$\frac{\triangle XYZ}{\triangle ABC} = \frac{\triangle XYZ}{\triangle ABC}$$

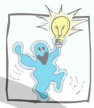
$$\text{Enlargement} = \frac{YZ}{BC}$$



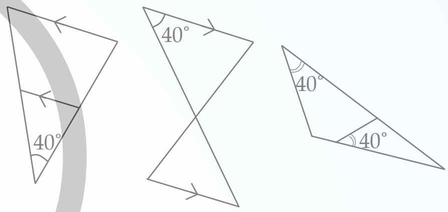


You can build up a list of mathematical definitions.

Similar triangle are equiangular triangles. One therefore is an enlargement of the other.

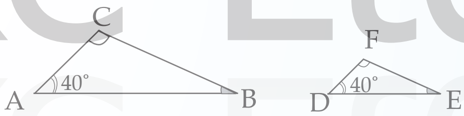


The pair of similar triangles are often combined together. Each of the following contains a pair of similar triangles.



You should identify the two equiangular triangles and you should draw them as two separate triangles as an aid to solving the problem. Then you can identify which sides are in the same relative position.

If you write the second triangle in the same relative order as the first triangle then it is easier to identify equivalent sides.



$$\text{Similar triangles} = \frac{\triangle ABC}{\triangle DEF}$$

Similarly we know side XY. X is above A and Y is above B so the second pair XY is an enlargement of AB.

$$\frac{YZ}{BC} = \frac{XY}{AB}$$

The other sides are selected because they are known lengths. We substitute in for each of our sides, YZ = x, BC = 8, XY = 32 and AB = 20.

$$\frac{YZ}{BC} = \frac{XY}{AB}$$

$$\frac{x}{8} = \frac{32}{20}$$

$$x = \frac{8 \times 32}{20}$$

$$x = 12.8 \text{ cm}$$

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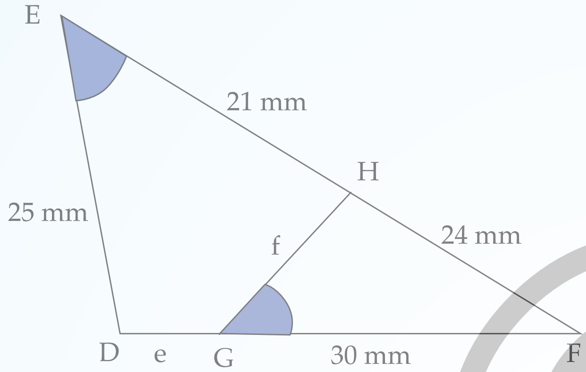
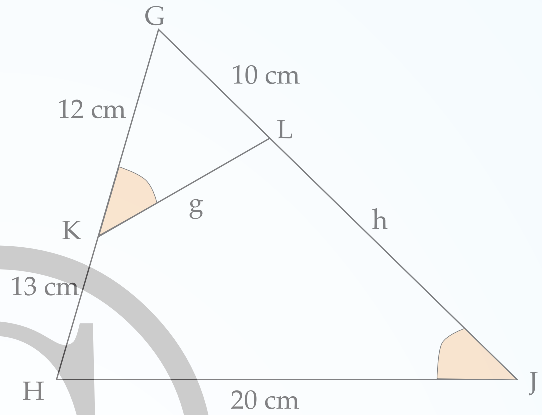
73. Identify, with justification, a pair of similar triangles and use these triangles to identify the lengths e and f. Angle DEF = Angle HGF (shaded).



74. Identify, with justification, a pair of similar triangles and use these triangles to identify the lengths g and h. Angle GKL = Angle GJH (shaded).



Excellence – Devise a strategy to investigate or solve a problem.

75. Form a model and solve it, justifying all steps. When Aaron walked exactly 12 metres from a lamp post his shadow had grown to 3.2 metres. If Aaron is 1.80 metres tall, what is the height of the lamp post?

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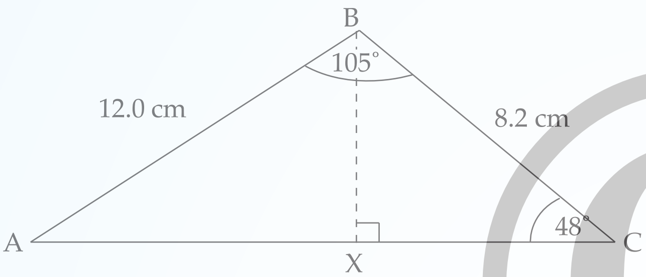
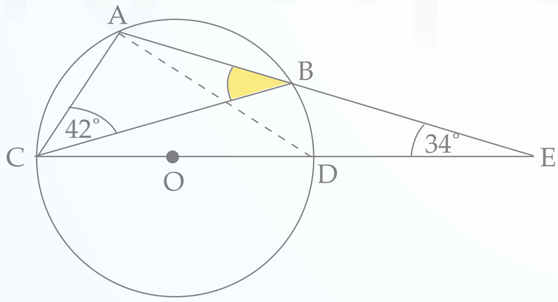
Mixed Problems

Merit / Excellence – Answer the following mixed problems.

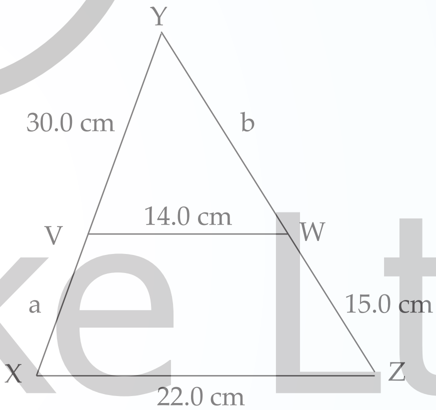
144. Find the length AC of the triangle ABC below, by first dividing the triangle into two smaller right-angled triangles. Show all your working,



145. In a circle chord AB and a diameter CD are extended outside the circle to meet at a point E. If ACB = 42° and AEC = 34° find angle ABC. Show all your working giving appropriate geometric reasons.



146. XY and ZY are transversals that cut the two parallel lines VW and XZ. Find the missing lengths a and b. Show all your working.



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The Statistical Enquiry Cycle

The Statistical Enquiry Cycle

The Statistical Enquiry Cycle is a model to enable a student to organise how to study perceived differences in a population. It organises how you will gather the evidence, analyse it and draw conclusions. EAS 1.12 Chance and Data requires you to evaluate statistical investigations or probability activities undertaken by others, including data collection methods, choice of measures, and the validity of findings. Also you will need to evaluate statistical reports in the media by relating the displays, statistics, processes, and probabilities used to the claims made, so being familiar with the statistical enquiry cycle is important.

Problem

Start with your Purpose Statement or what you expect to achieve with this investigation.

Then specify a statistical question as to what you will investigate about your population. A statistical problem will involve a comparison and require evidence in order to infer a possible answer. "I wonder if ...".

Plan

The plan describes how you will collect the data you need as evidence.

- What is the population you will investigate?
- What data do you need and how will you collect it?
- How will you get a random sample from the population? What size sample will you take?
- How will you record the information?

Data

This is where your data is randomly collected, managed and organised.

It should be collected in such a way that it is easy to extract information from. Sometimes the collection and managing is done for you by a computer programme.

You may need to clean the data.

Analysis

This is where you look at the data to see what it tells you about your problem.

Sort the data if required and collect statistics on the subsets of the population. The results should be displayed to best show differences and similarities between the subsets you wish to compare.

Conclusion

Have you enough evidence to answer your question?

- What do the statistics and displays tell you about the data and distributions?
- Are the differences in your sample sufficient for you to infer that there are differences in the population?
- Does the investigation suggest new problems that could be investigated?

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Excellence – Demonstrate your understanding of chance, justifying statements and findings. This may involve you critically reflecting on the validity of results and conclusions.

The data in the table is extracted from the National Year 10 ASH Snapshot Survey. The ASH Year 10 survey is a national census style survey that has been conducted annually since 1999. Over 330,147 New Zealand fourteen and fifteen year old teenagers have completed the survey since 1999. The survey focuses on smoking by teenagers, their family and friends.

Year	1999	2004	2009
Daily*	4 529	3 128	1 443
Weekly*	1 945	1 277	696
Monthly*	1 829	1 213	670
Regular (=*)	8 303	5 618	2 809
Less than Monthly	4 152	2 458	1 340
Experimented	7 403	8 842	5 127
Never Smoked	9 174	15 003	16 488
Sample total (N)	29 032	31 921	25 764

*Is the combined total of students who report smoking daily, weekly or monthly.

Source: National Year 10 ASH Snapshot Survey, 1999-2009: Trends in Tobacco Use by Students Aged 14-15 Years.

29. In 2009 ASH surveyed 25 764 fourteen and fifteen year olds. Use this data to investigate the probability of smoking among the New Zealand population of Year 10 students.

a) There were 25 764 students involved in the 2009 survey. If a student is selected at random from this group, what is the probability that they have smoked at least once?

b) In part a) the term 'have smoked' was used. What are the likely problems with the definition of the description 'have smoked' in describing the students who smoke?

c) How many more times likely was a fourteen and fifteen year old teenager in 1999 likely to smoke daily than in 2009?

ASH uses a definition of a regular smoker as a fourteen and fifteen year old who reports smoking daily, weekly or monthly.

d) Is this a good definition? Improve upon this definition of 'Regular smoker' explaining why your definition is a better. Calculate the probability of a student being a 'regular' smoker by your definition in 1999, 2004 and 2009.

There has been a trend over the 10 years for the consumption of cigarettes by fourteen and fifteen year old teenagers to decrease.

e) Which category of smokers has shown the smallest percentage decrease? Justify your answer.

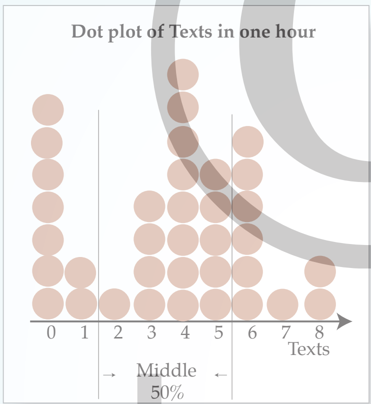
f) Why do you think the category identified in your answer in e) has decreased less than the other categories?

Display of Data

Dot Plots

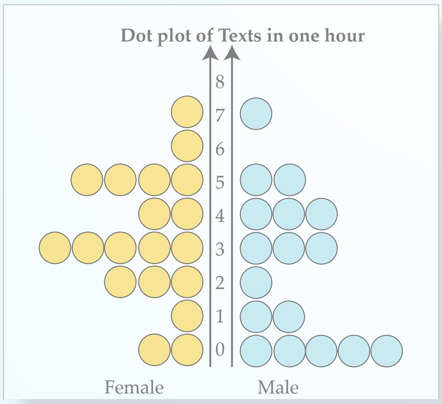
A **Dot** plot is used particularly with numbers over a limited range. Individual data points are represented as dots on the graph. Dot plots are one of the simplest statistical plots, and are suitable for numerical data sets. They are useful for highlighting clusters and gaps, as well as extreme values or outliers.

To use your dot plot to show similarities and/or differences between samples it is important to mark on the dot plot the distribution of the middle 50% of data. With 36 pieces of data the middle 50% of data excludes the nine pieces of data at each end of the distribution, so if you lightly cross these 18 pieces of data off you are left with the middle 50% and you would mark this on your graph.



The spike of data of zero texts in one hour is immediately apparent with this dot plot.

Although it is possible to plot rounded continuous data on a dot plot it is usually used for counted data. When dealing with larger data sets (over 30) the related stem and leaf plot or histogram may be more efficient, as dot plots may become too cluttered after this point. For two samples from one population the dot plots are normally placed side-by-side to show differences in the distribution.



When we split our dot plot into male and female responses we can see differences in the distribution for each sample of our population.

Stem and Leaf Plots

Stem and Leaf plots are used when collecting data as they give us an ongoing 'picture' of the data and its shape. They are also an effective tool in organising data and they simplify the calculation of the median and quartiles. They also give an indication of the distribution (as a stem and leaf graph is similar to a histogram).

To draw a stem and leaf plot we begin by creating a **stem** usually made up of one or two significant figures of the data values.

The leaves are made up of the remaining significant figures. A set of leaves from the stem is called a **branch**. Select the stem so that there are between 5 and 8 branches if possible. Data is added as it is collected so initially the data in each branch is not ordered.

Stem and Leaf Plot
Not ordered

6	0													
5	1	8	7	7	1									
4	5	1	6	6	4	4	3	2	4	0	2	3	5	
3	9	0	7	3	2	8	2	8	6	9	2	4		
2	0	5	1	1	5	6	5	7	6	2	5	9	6	9
1	8	9	8	8										

Each branch is then ordered so quartiles can be calculated. With 49 pieces of data the median is the $(49 + 1)/2 = 25$th position. We ignore the median, if it is an actual value, in calculating the quartiles. The quartiles are $(24 + 1)/2 = 12.5$ (between 12th and 13th position) from each end as there are 24 data values either side of the median.

Stem and Leaf Plot
Ordered

6	0													
5	1	1	7	7	8									
4	0	1	2	2	3	3	4	4	4	5	5	6	6	
3	0	2	2	2	3	4	6	7	8	8	9	9		
2	0	1	1	2	5	5	5	5	6	6	6	7	9	9
1	8	8	8	9										

Median is in position 25 and quartiles are between the 12th and 13th from each end.

If we are collecting data from two sources, e.g. male and female, we can represent the data on a back-to-back stem and leaf plot. A visual comparison between the two sets of data is then possible.

Stem and Leaf Plot

Female										Male						
									6	0						
									5	1	1	7	7	8		
			6	5	4	4	3	2	4	0	1	2	3	4	5	6
			7	6	2	2	2	0	3	3	4	8	8	9	9	
9	7	6	5	5	5	1	1	0	2	2	5	6	6	9		
					9	8	8	8	1							

77. The table below gives the net permanent and long-term net migration figures to and from New Zealand for the period 2001 to 2010.
(*Source: Statistics NZ website*)

Year	2001	2002	2003	2004	2005
Migration (000)	$^{-}11$	28	42	25	9
Year	2006	2007	2008	2009	2010
Migration (000)	10	11	5	9	20

a) Draw a bar graph to represent the data on the axes.

b) Explain what the negative value in the table represents.

c) Over the 10 year period what has been the net gain in terms of population for New Zealand?

d) Describe the trend in migration figures over the 10 years.

e) What are possible causes to changes in net migration figures over the 10 years?

78. The table below gives the short term visitors to New Zealand for the year ended September 2007.
(*Source: Statistics NZ website*)

Country	Aust.	China	Japan	Korea
Visitors (000)	940	141	126	108
Country	UK	USA	Other	Total
Visitors (000)	302	218	630	2 465

a) Draw a bar graph to represent the data on the axes.

b) Australia's population is about 20 million. What percentage of the Australian population visited New Zealand in this period assuming every visitor came only once?

c) New Zealand's population is 4.1 million. One commentator after looking at the figures, stated that 'over a third of people in New Zealand are visitors'. What is wrong with this statement?

d) Looking at these figures, if you had to advise on a major tourist advertising campaign, which of the countries has the most potential to increase their tourist numbers to New Zealand? Justify your choice.

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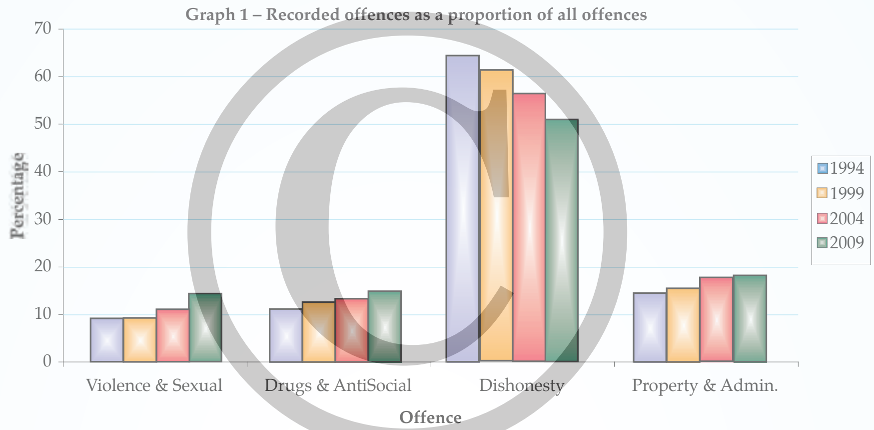
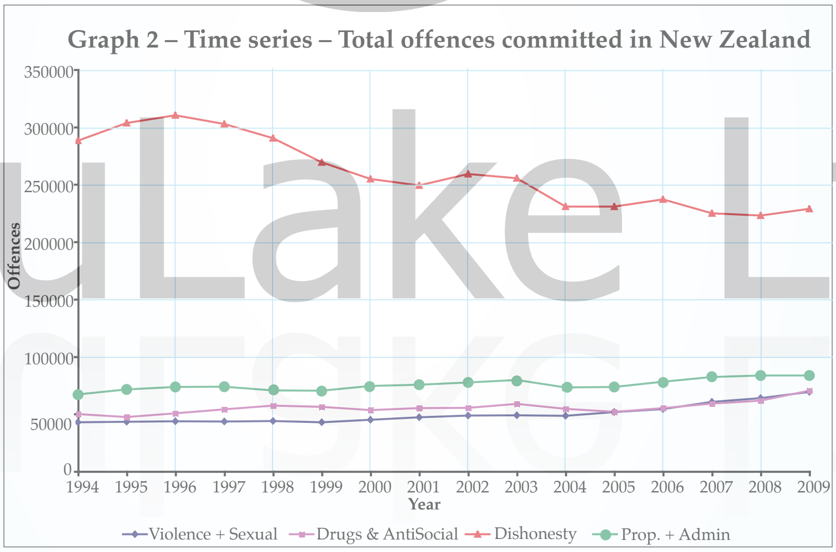
Practice External Assessment Task

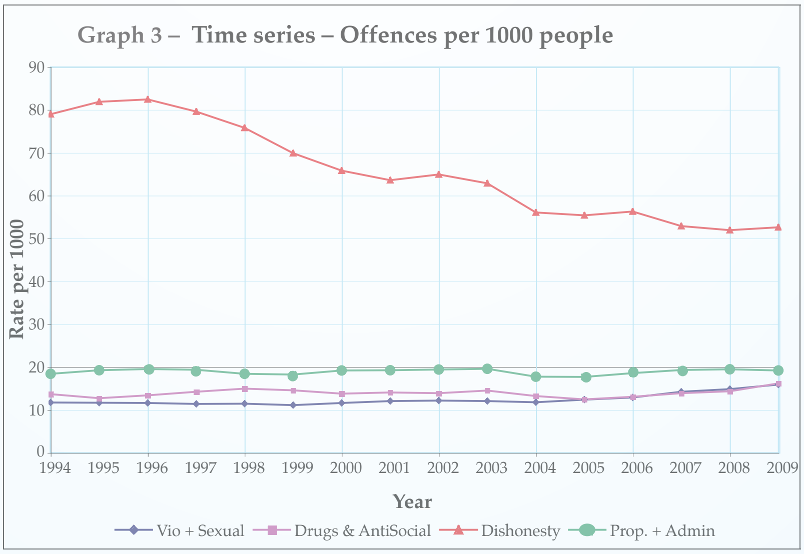
Chance and Data 1.12

QUESTION ONE

Study the information in the graphs below and on the following page, with regard to criminal offences in New Zealand during the period 1994 – 2009, then answer the questions that follow. (*Source: Statistics New Zealand website)*







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Page 42 cont...

356. $\frac{x-3}{x+7}$

Page 43

357. $\frac{x}{x+5}$

358. $\frac{2(x+3)}{x-1}$ or $\frac{2x+6}{x-1}$

359. $\frac{1}{3(x-2)}$

360. $\frac{k+9}{k+2}$

361. $\frac{2(m+3)}{m+5}$

362. $\frac{q}{q+2}$

363. $\frac{r+3}{r-5}$

364. $\frac{p-4}{p+2}$

365. $\frac{2(x+4)}{3(x-2)}$

366. $\frac{x+5}{x-5}$

Page 46

367. $x = 1$, $y = 3$

368. $x = 3$, $y = {}^{-}2$

369. $x = 2$, $y = 6$

370. $x = {}^{-}3$, $y = 1$

371. $x = {}^{-}5$, $y = {}^{-}1$

372. $x = 2$, $y = {}^{-}3$

373. $x = {}^{-}1$, $y = 1$

374. $x = 2$, $y = {}^{-}1$

Page 47

375. $x = 1$, $y = 2$

376. $x = 3.5$, $y = 2$

Page 47 cont...

377. $x = 3$, $y = {}^{-}3$

378. $x = 1$, $y = 3$

379. $x = 2$, $y = 3$

380. $x = 2$, $y = 1$

381. $x = 4$, $y = 5$

382. $x = 0$, $y = 4$

Page 49

383. $x = {}^{-}3$, $y = 1$

384. $x = {}^{-}13$, $y = {}^{-}10$

385. $x = {}^{-}3$, $y = 1$

386. $x = {}^{-}2$, $y = 2$

387. $x = {}^{-}5$, $y = {}^{-}1$

388. $x = {}^{-}4$, $y = {}^{-}4$

389. $x = 5$, $y = 2$

390. $x = 1$, $y = 4$

Page 50

391. $x = 3$, $y = 0$

392. $x = 2$, $y = 5$

393. $x = {}^{-}2$, $y = {}^{-}4$

394. $x = 0$, $y = 3$

395. $x = 3$, $y = {}^{-}1$

396. $x = 5$, $y = 3$

397. $x = 0$, $y = {}^{-}4$

398. $x = {}^{-}4$, $y = 1$

Page 51

399. Flour \$2 kg
Eggs \$6 dozen

400. Larger number is 36
Smaller number is 15

Page 52

401. Grandstand \$40
Terraces \$15

402. Width 28 cm
Length 112 cm

403. Calculus \$20
Year 11 \$25

404. Adults 300
Students 200

405. \$3 ice-creams 35
\$5 ice-creams 15

406. 15 two-mark questions.
20 three-mark questions.

Page 53

407. $2b + d = 15$, $b + 2d = 12$
Burger = \$6, dessert = \$3.

408. $25x + 3y = 20$, $10x + 5y = 27$
1(00) texts and 15 min. = \$8

409. a) $9x + 3y = 300$
b) $6x + 8y = 440$
c) 20 kg poultry,
40 kg compost.

410. a) $2000A + 8000B = 34000$
$4000A + 2000B = 26000$
b) $A = \$5$, $B = \$3$

Page 54

411. $3x + 15 = 27$

412. $200 - 5x = 30$

413. $x + (x + 1) + (x + 2) = 60$
$3x + 3 = 60$

414. $x^2 + 16 = 20$

415. $x - 5$

416. $3k + 3$

Page 55

417. $x + 8x = 423$
$x = 47$
Travelled 47 km by car

418. $2x \times 7 = 182$
$x = 13$

419. $x(x + 2) = 48$
$x = 6$ Disregard $x = {}^{-}8$
Dimensions 6 cm x 8 cm.

420. $x(x + 10) = 75$
$x = 5$ Disregard $x = {}^{-}15$
Dimensions 5 m x 15 m.

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