

Year 13

Mathematics

IAS 3.4

Critical Path Analysis

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Contents

•	Achievement Standard	2
•	Precedence Tables	3
•	Network Diagrams	9
•	Dummy Activities	20
•	Critical Path Algorithm	28
•	Float Times	38
•	Scheduling	46
•	Practice Internal Assessment	58
•	Answers	63
•	Order Form	71

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graph LR; 0((0)) -- A --> 1((1)); 1 -- B --> 2((2)); 1 -- C --> 3((3)); 2 -- D --> 3; 2 -- E --> 4((4)); 3 -- F --> 5((5)); 4 -- G --> 5; 5 -- H --> 6((6));
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NCEA 3 Internal Achievement Standard 3.4 – Critical Path Analysis

This achievement standard involves using critical path analysis in solving problems.

Achievement	Achievement with Merit	Achievement with Excellence
<ul style="list-style-type: none"> Use critical path analysis in solving problems. 	<ul style="list-style-type: none"> Use critical path analysis with relational thinking, in solving problems. 	<ul style="list-style-type: none"> Use critical path analysis with extended abstract thinking, in solving problems.

- ◆ This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives
 - ❖ develop network diagrams to find optimal solutions, including critical paths in the Mathematics strand of the Mathematics and Statistics Learning Area.
- ◆ Use critical path analysis in solving problems involves:
 - ❖ selecting and using methods
 - ❖ demonstrating knowledge of concepts and terms
 - ❖ communicating using appropriate representations.
- ◆ Relational thinking involves one or more of:
 - ❖ carrying out a logical sequence of steps
 - ❖ connecting different concepts or representations
 - ❖ demonstrating understanding of concepts
 - ❖ forming and using a model;
 and relating findings to a context, or communicating thinking using appropriate mathematical statements.
- ◆ Extended abstract thinking involves one or more of:
 - ❖ devising a strategy to investigate or solve a problem
 - ❖ identifying relevant concepts in context
 - ❖ developing a chain of logical reasoning, or proof
 - ❖ forming a generalisation;
 and using correct mathematical statements, or communicating mathematical insight.
- ◆ Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts.
- ◆ Methods include a selection from those related to:
 - ❖ precedence tables
 - ❖ network diagrams
 - ❖ critical events
 - ❖ scheduling
 - ❖ float times.

Critical Path Analysis



Critical Path Analysis

In this Achievement Standard we focus on the modelling of procedures or projects.

This process of modelling is often called Network Analysis or Critical Path Analysis and involves the use of specific techniques for planning, scheduling, management and control of a project.

Critical Path Analysis enables us to break the project down into individual components or activities and then record the results in an appropriate network diagram.

Critical Path Analysis is a valuable management tool because it results in

- a clear identification of all major components or activities in a project.
- a logical sequencing of all components or activities.
- an allocation of time required for each component or activity.
- a scheduling of the activities or components in the most efficient manner.
- a visual picture in the way of a network diagram of the project
- the ability to evaluate the network in an ongoing manner.

Precedence Tables

One of the first steps in scheduling a project is to break it down into a series of components or activities and represent these in table form.

Such a table is called a Precedence table (or Dependence table) and it lists all activities required to complete a project with an applicable time (duration) alongside each activity. Included as well are any dependencies, i.e. when an activity depends on another being completed first, it is included in the table.

Study the Precedence table drawn on the right for the 'Construction of a Garden Shed'.

The column headed Precedence ('Depends on') shows those activities that have to be completed before a particular one can be started. For example, the walls of the garden shed cannot be erected before the site is cleared, the foundations have been laid and the timber purchased, hence activity D has B and C in the precedence column alongside it.

The precedence column only shows the activities immediately preceding each entry in the activity column, but by implication these can include other activities.



This activity can be done without having to do any other activity before it.

This is the time (duration) the activity takes.

Activity and Description (Garden shed construction)	Precedence (Depends on)	Duration (days)
A Clear building site	–	1
B Lay foundations	A	4
C Purchase timber	–	1
D Erect walls	B, C	3
E Construct roof	D	2
F Clad Walls	E	3
G Install door and windows	F	1
H Fit spouting	G	1
I Paint	H	2

B, C in the Precedence column means that before the walls can be erected the foundations have to be laid and the timber purchased.

For example D depends on B and C, and in turn B depends on A.

So in order to erect the walls of the garden shed the site has to be cleared, the foundations laid and the timber purchased.

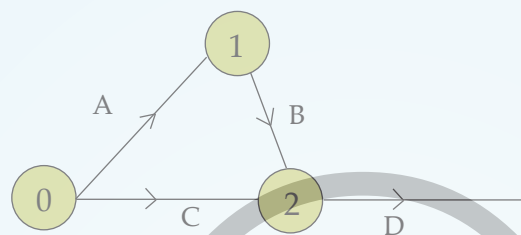
Consider cladding the walls of the garden shed which is F.

F depends on E, but since E depends on D it is implied that F depends on D. We already know that D depends on B and C and in turn B depends on A.



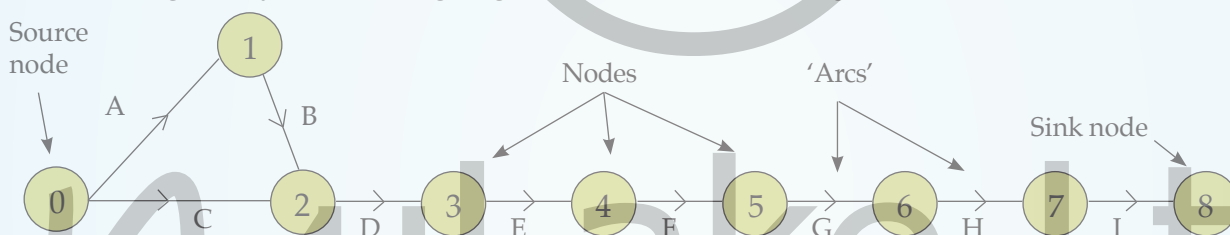
Network Diagrams cont...

- Next activities B and C must be completed before activity D, 'Erect walls' can be started. Therefore we insert node 2, which indicates the completion of activities B and C and add activity D.



Sometimes your first attempt at drawing a network diagram may not be perfect so it may be necessary to adjust it as you go. You can see above that we have had to adjust the shape of our network diagram in order to add activity D into the network diagram.

- The rest of the network diagram for the 'Construction of a shed' is straightforward as each remaining activity only depends on its predecessor, i.e. activity E depends on activity D, activity F depends on activity E, activity G depends on activity F, activity H depends on activity G and finally activity I depends on activity H. We can represent this by just adding each remaining activity to the existing diagram. Our final network diagram can be seen below.

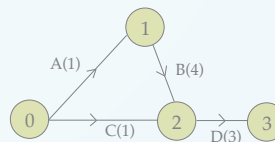


Once all the activities have been undertaken the project is complete. We add a finishing node (called a sink node), node 8 to indicate this.

Activity and Description (Garden shed construction)	Precedence (‘Depends on’)	Duration (days)
A Clear building site	–	1
B Lay foundations	A	4
C Purchase timber	–	1
D Erect walls	B, C	3
E Construct roof	D	2
F Clad Walls	E	3
G Install door and windows	F	1
H Fit spouting	G	1
I Paint	H	2

On an activity network the activities are represented by lines ('arcs') and the weight on each line is the duration of the activity.

On our activity network above we have not included the duration, but will do so in the next section. When we do this we will include the duration in brackets e.g. A(1), B(4), C(1), D(3) etc.



The nodes (or vertices) on an activity network represent events, where each event represents the completion of one or more activities.



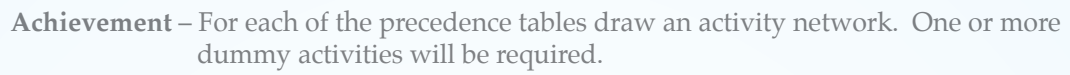
When we construct an activity network, convention is that we construct it from left to right.



The source node represents the start of a project and the sink node the end of the project.



An activity network consists of nodes and arcs. The nodes are represented by circles and the 'arcs' by lines between the nodes. A node indicates the completion of one or more activities.



Activity	Precedence ('Depends on')
A	—
B	—
C	A
D	A
E	B
F	B
G	C, D
H	E, F
I	G, H
J	C
K	F
L	I, J, K

Activity	Precedence (‘Depends on’)
A	–
B	–
C	A, B
D	B
E	C, D
F	D
G	C, D
H	E, G
I	F, H

**Example**

A precedence table with duration times (hours) for the activities required to complete a project is given below. Draw up an activity network to model this project and then undertake a forward and backward pass of the activity network.

Activity	Precedence ('Depends on')	Duration (hours)
A	–	10
B	–	15
C	A	20
D	A	5
E	B	10
F	B	15
G	D, E	15
H	D, E	10
I	C, G	5
J	F, H	15
K	I, J	25

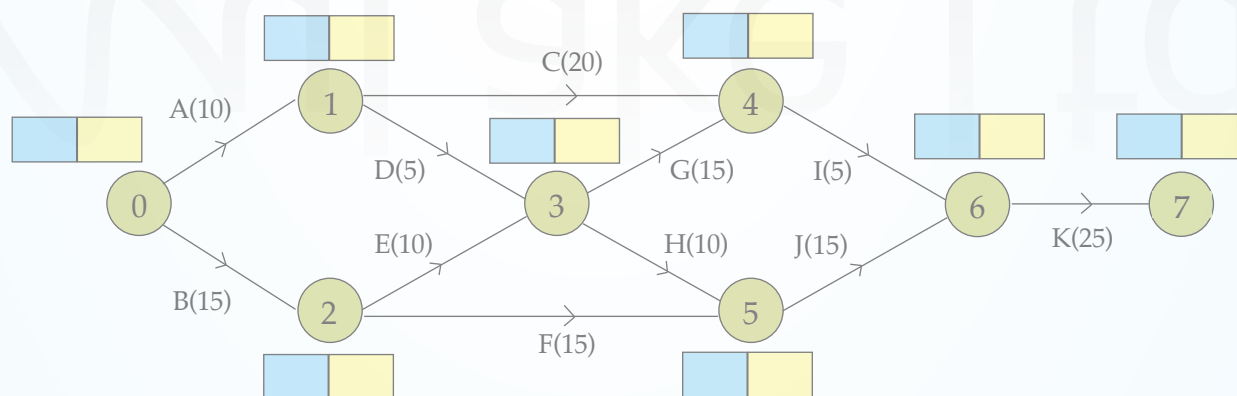


When undertaking a forward pass remember you want the latest start time (LST).

Put your finger on a node and ask yourself what is the LST from the nodes that point to the node with your finger on.



We begin by drawing up the activity network for this precedence table. Note, we have included the duration times in brackets after each activity and have included a box alongside each node so we can enter in the earliest and latest event times when we undertake our forward and backward pass.



We now begin our **forward pass** (blue) of the activity network.

At node 0 we put the duration time of 0 in the left hand rectangle (blue).

At node 1 the duration time must be 10 as it takes 10 hours to complete this activity.

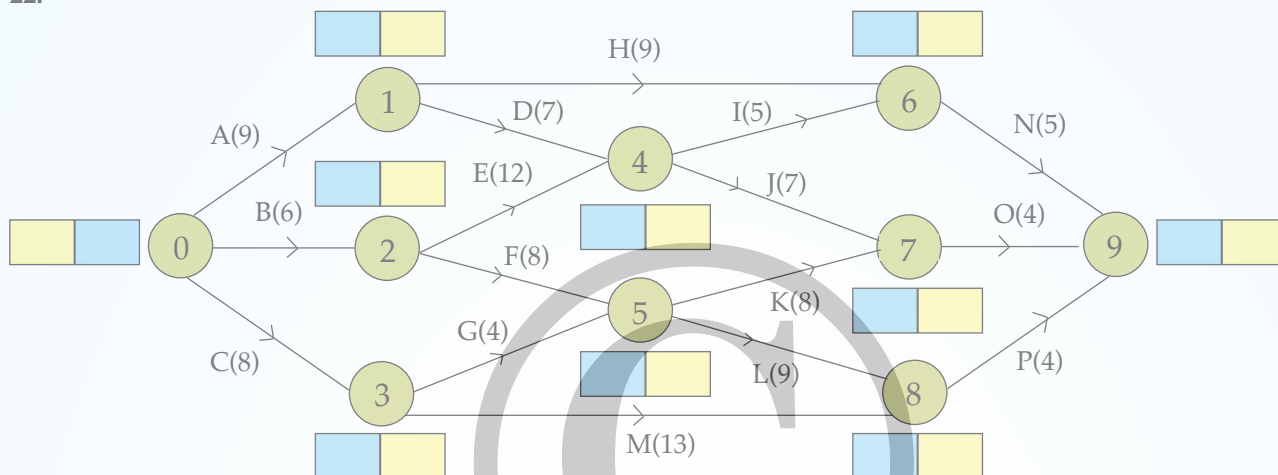
At node 2 the duration time must be 15 as it takes 15 hours to complete this activity.

At node 3 we have two possibilities which we must calculate. The path 0 – 1 – 3 which gives a total of 15 hours and the path 0 – 2 – 3 which gives a total of 25 hours. The maximum of these two values is 25 which is the value we enter for node 3.

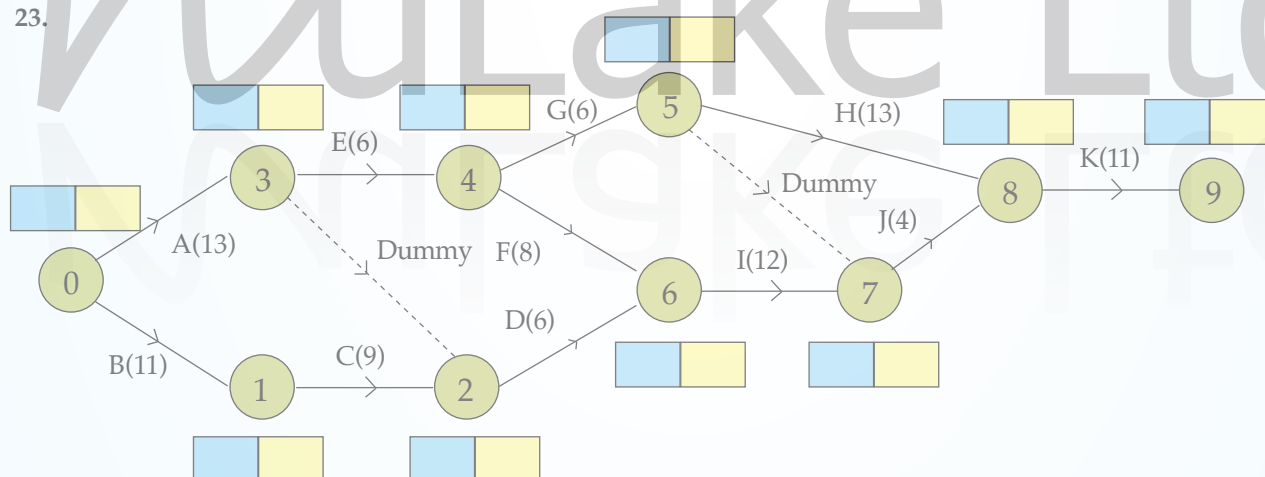


Achievement – For each of the activity networks below undertake a forward and backward pass to calculate the early and late times for each event. Determine the critical events and the critical path of the network.

22.



23.



The network above includes two dummy activities. Remember that dummy activities have a duration of zero.

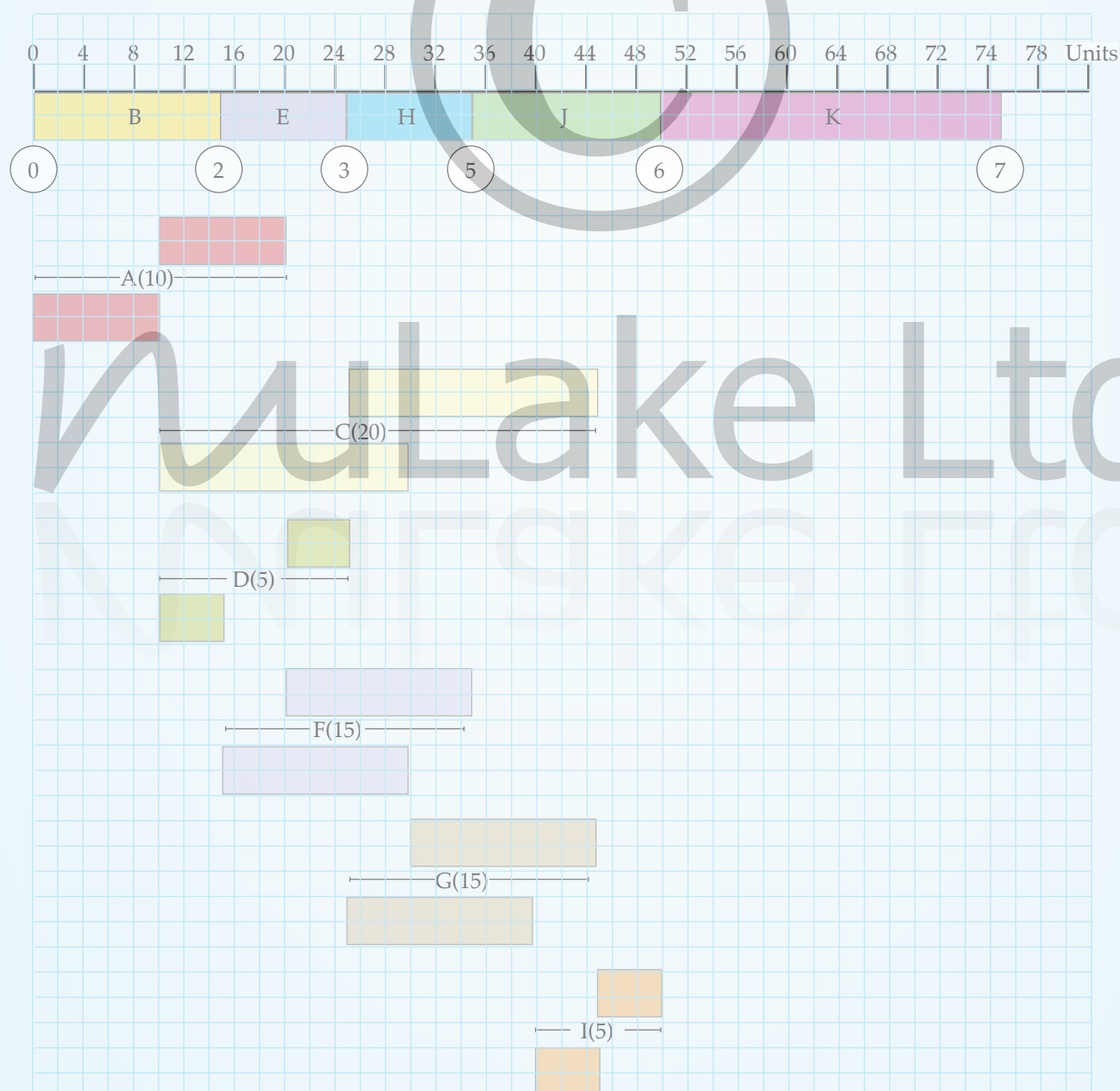


Scheduling cont...

Things to note about the Gantt chart drawn below.

- Time units are listed across the top from 0 to 75 as the project is complete after 75 time units.
- All critical activities are positioned first in the correct order. Each critical activity is labelled and the critical events are marked at the junction of the critical activities.
- Each non-critical activity is taken in turn. For example, activity A has a duration of 10 time units and has an EST of 0 and a LFT of 20. This non-critical activity of 10 time units can fit in anywhere between 0 and 20 time units. Hence two bars are drawn each of 10 time units, one starting at the EST and the other finishing at the LFT.

Gantt Chart



Answers

Page 6 (Other answers possible)

1.	Activity and Description (Bake a cake)	Precedence (‘Depends on’)	Duration (mins.)
A	Select recipe	–	10
B	Preheat the oven	A	15
C	Find and grease baking...	–	2
D	Get ingredients	A	5
E	Measure and mix ingre...	D	10
F	Pour mixture into tin	C, E	1
G	Bake cake in oven	B, F	25
H	Place cake on tray to cool	G	45

2.

Activity	Precedence (‘Depends on’)
A	–
B	–
C	A
D	B, C
E	–
F	D, E

3.	Activity and Description (Make breakfast)	Precedence (‘Depends on’)	Duration (mins.)
A	Get up and dressed	–	15
D	Fill kettle with water	A	1
F	Boil the kettle	D	3
E	Make the tea	F	2
C	Get bread	A	1
H	Toast bread	C	3
G	Put spread on toast	H	1
B	Eat toast and drink tea	E, G	20

Page 7

4.

Activity	Precedence (‘Depends on’)
A	–
B	–
C	–
D	B
E	A
F	A
G	B
H	C, D
I	E
J	E
K	F, G, I
L	K, H, J

5.

Activity	Precedence (‘Depends on’)
A	–
B	–
C	–
D	A
E	A
F	B
G	B
H	C, D, E, F
I	E
J	E
K	I
L	I
M	G, H, J, K, L

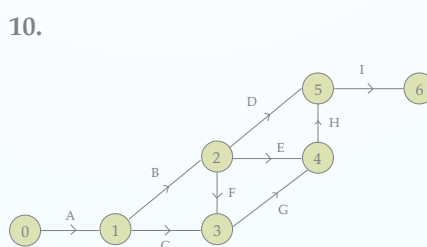
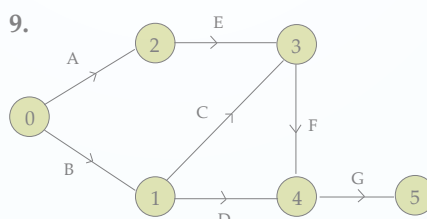
Page 8 (Other answers possible)

6.	Activity and Description (Putting on a play)	Precedence (‘Depends on’)	Duration (weeks.)
A	Choose play	–	1
B	Print tickets	A	3
C	Sell tickets	B	4
D	Choose cast	A	2
E	Hand out scripts to cast	D	1
F	Rehearsals	E	4
G	Make cast wardrobe	D	3
H	Organise sound, props, lights	F	1
I	Dress rehearsal	G, H	3
J	Play season starts	C, I	1

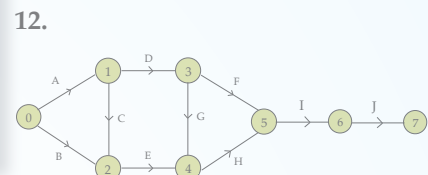
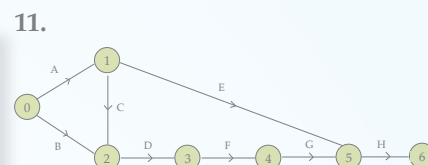
7.	Activity and Description (Wooden deck chair)	Precedence (‘Depends on’)	Duration (hours.)
A	Designs the chair	–	1
B	Purchases materials required	A	1
C	Makes the seat of the chair	B	2
D	Makes the back of the chair	B	1
E	Makes the legs of the chair	B	2
F	Cuts foam for cushion	B	0.5
G	Covers foam for cushion	F	1
H	Assembles chair	C, D, E	2
I	Stains chair	H	2
J	Fixes cushion to chair	G, I	1

8.	Activity and Description (Overseas holiday)	Precedence (‘Depends on’)	Duration (Hours)
A	Decide time and destination of holiday	–	2
B	Book flights	A	0.5
C	Book accommodation	B	0.5
D	Arrange travel documents	C	1
E	Arrange currency	D	1
F	Book transfers to/from accommodation	C	0.25
G	Organise taxi to/from airport	B	0.25
H	Book Tours	C	0.5
I	Pack Bags	E, G, H	2

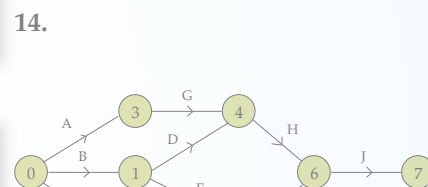
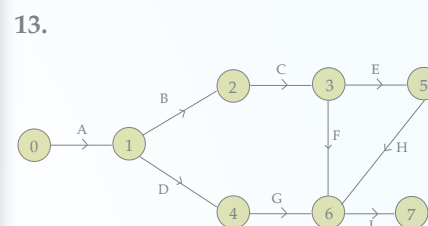
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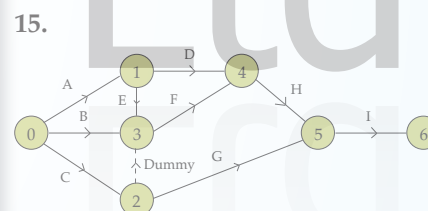
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Page 25 (Other layouts possible)



Page 26 (Other layouts possible)

